SEMESTER–I

PERIOD–I

M12CH1

TOPIC 1

Sequence and Series

1.1. SEQUENCES

A succession of numbers formed according to a certain rule and arranged in a definite order is called a **sequence**.

Illustration. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots, \frac{1}{2n}$ ... is a sequence.

In a sequence, the numbers occurring at its first place, second place, third place, ... *n*th place are respectively called its first term, second term, third term, ..., *n*th term.

1.2. ARITHMETIC PROGRESSIONS

A sequence is said to be an **arithmetic progression (AP)** if the difference of each term, except the first one, from its preceding term is always same.

For example 2, 5, 8, 11, ... is an AP, because

$$5 - 2 = 3,\ 8 - 5 = 3,\ 11 - 8 = 3, \ldots$$

Thus, the sequence $\{T_n\}$ is an arithmetic progression, if there exists a number, say, d such that $T_{n+1} - T_n = d$ for $n \geq 1$.

Definition of an AP

If '*a*' and '*d*' be the first term and common difference of the AP $\{T_n\}$.

$$T_n = a + (n-1)d, \quad n \in \mathbf{N}.$$

Example 1. *Find the number of terms in the Arithmetic progression (A.P.) 7, 10, 13, ..., 31.*

Solution. Given A.P. is 7, 10, 13, ..., 31

Since first term $a = 7$, common difference $d = 10 - 7 = 3$ and $T_n = 31$.

We know that $T_n = a + (n-1)d$

$$31 = 7 + (n-1)3$$
$$31 - 7 = 3n - 3$$
$$24 = 3n - 3$$
$$3n = 27$$
$$n = 9$$

Hence, the number of terms in the A.P. is 9.

Example 2. *Find the sum of the following arithmetic progression 9, 15, 21, 27, ... The total number of term is 14.*

Solution. Given A.P. is 9, 15, 21, 27, ...

Here first term $a = 9$, common difference $d = 15 - 9 = 6$

Number of term $n = 14$.

We know that $S_n = \frac{n}{2}[2a + (n-1)d$

$$S_{14} = \frac{14}{2}[2 \times 9 + (14-1)6]$$
$$= 7[18 + 78] = 7 \times 96 = 672$$

Hence, the sum of the given A.P. is 672.

Example 3. *Find the m^{th} term of an A.P. sum of whose first n terms is $2n + 3n^2$.*

Solution. Given that sum of first n terms of an A.P. is $2n + 3n^2$

i.e., $S_n = 2n + 3n^2$

Hence the m^{th} term of the A.P. is

$$T_m = S_m - S_{m-1}$$
$$= (2m + 3m^2) - \{2(m-1) + 3(m-1)^2\}$$
$$= (2m + 3m^2) - \{(2m-2) + 3(m^2 + 1 - 2m)\}$$
$$= (2m + 3m^2) - (2m - 2 + 3m^2 + 3 - 6m)$$
$$= 2m + 3m^2 - 2m + 2 - 3m^2 - 3 + 6m$$
$$= 6m - 1.$$

Example 4. *Find the 17^{th} term from the end of the A.P. 1, 6, 11, 16, ..., 21, 216.*

Solution. Given A.P. is 1, 6, 11, 16, ..., 211, 216.

Here $a = 1$, $d = 6 - 1 = 5$, and last term $l = 216$, $n = 17$

We know that: $T_n = l - (n - 1)d$

$$T_{17} = 216 - (17 - 1)5$$
$$= 216 - 80 = 136.$$

1.3. GEOMETRIC PROGRESSIONS

A sequence of *non-zero* numbers is said to be a **geometric progression** (**GP**) if the ratio of each term, except the first one, by its preceding term is always same.

For example, 3, 6, 12, 24, ... is a GP, because

$$\frac{6}{3} = 2, \frac{12}{6} = 2, \frac{24}{12} = 2, \cdots$$

Thus, the sequence $\{T_n\}$ with $T_n \neq 0$ is a geometric progression if there exists a non-zero number, say, r such that $\frac{T_{n+1}}{T_n} = r$ for $n \geq 1$.

The constant number 'r' mentioned above is called the **common ratio** of the corresponding GP. The common ratio of a GP is denoted by 'r'.

The first term of a GP, is generally denoted by 'a'.

Definition of a GP

If 'a' and 'r' be respectively the first term and common ratio of the GP $\{T_n\}$, then

$$T_n = ar^{n-1}, \quad n \in \mathbf{N}. \qquad ...(1)$$

Example 5. *Find the 9th and nth terms of the sequence 3, 6, 12, 24, ...*

Solution. Given sequence is 3, 6, 12, 24, ...(1)

Here, $\frac{T_2}{T_1} = \frac{6}{3} = 2, \quad \frac{T_3}{T_2} = \frac{12}{6} = 2, \ldots \qquad \therefore \frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots = 2$

$\therefore$ (1) is a GP with $a = 3$ and $r = 2$.

Now, $T_9 = ar^{9-1} = ar^8 = 3(2)^8 = 3(256) = \mathbf{768}$ $[T_n = ar^{n-1}]$

and $T_n = ar^{n-1} = \mathbf{3(2)^{n-1}}$.

Example 6. *Which term of the series* $\frac{1}{4} - \frac{1}{2} + 1 + \ldots$ *is 256?*

Solution. The series is $\frac{1}{4} + \left(-\frac{1}{2}\right) + 1 + \ldots$

Here, $\frac{T_2}{T_1} = \frac{-1/2}{1/4} = -2, \quad \frac{T_3}{T_2} = \frac{1}{-1/2} = -2, \ldots$

$\therefore \quad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots = -2$

$\therefore$ The given series is a GS with $a = 1/4$ and $r = -2$.

Let $T_n = 256 \quad \therefore \quad ar^{n-1} = 256$

$\Rightarrow \quad = 256 \quad \Rightarrow \quad (-2)^{n-1} = 1024$

$\Rightarrow \quad (-2)^{n-1} = 210 \quad \Rightarrow \quad (-2)^{n-1} = (-2)^{10}$

$\Rightarrow \quad n - 1 = 10 \quad \Rightarrow \quad n = 11.$

$\therefore$ 256 is the **11th term.**

Example 7. *For what value of n, the nth terms of the series "5 + 10 + 20 + ..." and "1280 + 640 + 320 + ..." are equal?*

Solution. Ist series: We have 5 + 10 + 20 + ...

Here, $a_1 = 5$ and $r_1 = \frac{10}{5} = \frac{20}{10} = 2$

$\therefore \quad T_n = a_1 r_1^{\,n-1} = 5(2)^{n-1}$

IInd series: We have 1280 + 640 + 320 + ...

Here, $a_2 = 1280$ and $r_2 = \frac{640}{1280} = \frac{320}{640} = \frac{1}{2}$

$\therefore \quad T_n = a_2 r_2^{\,n-1} = 1280 \left(\frac{1}{2}\right)^{n-1}$

Sum of first *n* Terms of A GP

The sum of first n terms of a GP is denoted by S_n.

If $\{T_n\}$ is a GP, then we have

$$S_n = T_1 + T_2 + T_3 + \ldots + T_n, \; n \in \mathbf{N}.$$

For example, 2, 6, 18, 54, L is a GP and:

$$S_1 = 2, \quad S_2 = 2 + 6 = 8, \quad S_3 = 2 + 6 + 18 = 26 \text{ etc.}$$

Definition

If a and r be respectively the first term and common ratio of a GP, then the sum of first n terms of this GP is given by

$$S_n = \begin{cases} na & \text{if} \quad r = 1 \\ \dfrac{a(1-r^n)}{1-r} & \text{if} \quad r \neq 1. \end{cases}$$

Example 8. *Find the 20th term of the G.P.* $\frac{5}{5}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$.

Solution. Given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Here first term $\quad a = \frac{5}{2}$

Common ratio $\quad r = \dfrac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{4} \times \frac{2}{5} = \frac{1}{2}$

and $\quad n = 20$

We know that $\quad T_n = ar^{n-1}$

$$T_{20} = \frac{5}{2}\left(\frac{1}{2}\right)^{20-1} = \frac{5}{2} \times \left(\frac{1}{2}\right)^{19} = 2^{\frac{5}{20}}.$$

Example 9. *In a G.P.* $T_{10} = 9$ *and* $T_4 = 4$*, then find* T_7.

Solution. We know that $\quad T_n = ar^{n-1}$

$\therefore \quad T_{10} = 9 = ar^9 \Rightarrow ar^9 = 9 \quad \ldots(i)$

$T_4 = 4 = ar^3 \Rightarrow ar^3 = 4 \quad \ldots(ii)$

Multiply equation (*i*) and (*ii*), we get

$$(ar^9)(ar^3) = 9 \times 4$$

$$a^2r^{12} = 36$$

$$(ar^6)^2 = 36$$

$$ar^6 = 6$$

i.e., $\quad T_7 = 6 \quad [\because \ T_n = ar^{n-1}]$

Example 10. *Find the sum of 7 terms of the G.P. 3, 6, 12, ...*

Solution. Here $a = 3$, $r = \frac{6}{3} = 2$ and $n = 7$

We know that $S_n = a\left[\dfrac{r^n - 1}{r - 1}\right]$

$$\left[\text{If } r < 1 \text{ then use this formula } i.e., a\left[\frac{1 - r^n}{1 - r}\right]\right]$$

$$= 3\left[\frac{(2)^7 - 1}{2 - 1}\right] = \frac{3(128 - 1)}{1} = 3 \times 127 = 381.$$

EXERCISE

1. Find the 20th and *n*th term of the sequence 4, 9, 14, 19, ...

2. Show that the sequence: log a, $\log \dfrac{a^2}{b}$, $\log \dfrac{a^3}{b^2}$, ... is an AP.

3. Which term of the series 37 + 32 + 27 + 22 + ... is –103?

4. Determine the 1st term and the 40th term of the AP whose 7th term is 34 and 15th term is 74.

5. Find the sum of all 3-digit numbers which leave the remainder 1 when divided by 4.

6. Evaluate:

(i) $\dfrac{1}{9} + \dfrac{2}{9} + \dfrac{1}{3} + \ldots +$ 25 terms (ii) 5 + 13 + 21 + ... + 181.

7. How many terms of the sequence –12, –9, –6, –3, ... must be taken to make the sum 54?

8. The first term of a GP is 1. The sum of third and fifth terms is 90. Find the common ratio of the GP.

9. If the 5th term of a GP is 16 and the 10th term is 1/2, find the GP. Also, find its 15th term.

10. Determine the number of terms in the GP $\{T_n\}$ if $T_1 = 3$, $T_n = 96$ and $S_n = 189$.